



Original Articles

Sensory cue combination in children under 10 years of age

James Negen^{a,*}, Brittney Chere^b, Laura-Ashleigh Bird^a, Ellen Taylor^c, Hannah E. Roome^d,
Samantha Keenaghan^a, Lore Thaler^a, Marko Nardini^a

^a Department of Psychology, Durham University, Durham DH1 3LE, UK

^b Department of Psychological Sciences, Birkbeck, University of London, London WC1E 7HX, UK

^c Department of Experimental Psychology, Medical Sciences Division, University of Oxford, Oxford OX2 6GG, UK

^d Center for Learning and Memory, University of Texas at Austin, Austin, TX 78712, USA



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ABSTRACT

Cue combination occurs when two independent noisy perceptual estimates are merged together as a weighted average, creating a unified estimate that is more precise than either single estimate alone. Surprisingly, this effect has not been demonstrated compellingly in children under the age of 10 years, in contrast with the array of other multisensory skills that children show even in infancy. Instead, across a wide variety of studies, precision with both cues is no better than the best single cue – and sometimes worse. Here we provide the first consistent evidence of cue combination in children from 7 to 10 years old. Across three experiments, participants showed evidence of a bimodal precision advantage (Experiments 1a and 1b) and the majority were best-fit by a combining model (Experiment 2). The task was to localize a target horizontally with a binaural audio cue and a noisy visual cue in immersive virtual reality. Feedback was given as well, which could both (a) help participants judge how reliable each cue is and (b) help correct between-cue biases that might prevent cue combination. Crucially, our results show cue combination when feedback is only given on single cues – therefore, combination itself was not a strategy learned via feedback. We suggest that children at 7–10 years old are capable of cue combination in principle, but must have sufficient representations of reliabilities and biases in their own perceptual estimates as relevant to the task, which can be facilitated through task-specific feedback.

1. Introduction

Cue combination is when two independent noisy perceptual estimates are merged together as a weighted average, creating a unified estimate that is more precise than either single estimate alone (Ernst & Banks, 2002). This process is a key aspect of Bayesian Decision Theory (Maloney & Mamassian, 2009), allowing people to deal in a near-optimal way with the different sources of sensory noise involved in perceptual decisions. For example, when both looking at and holding an object, we have information about its size from vision as well as from touch. If we always take the approach of averaging the available estimates while weighting each independent estimate by its precision, we will always arrive at the most precise possible estimate of size. The main alternatives would be strategies such as cue selection, where one cue is used and the other is ignored, or more rote forms of integration like taking an unweighted average of the estimates.

Since a seminal report in 2002 (Ernst & Banks, 2002), evidence has accumulated that a Bayesian cue combination model is a good description of how adults typically perform when given multiple

independent cues to a singular perceptual judgement (Knill & Pouget, 2004; Pouget, Beck, Ma, & Latham, 2013). In particular, the strongest single test for cue combination is a bimodal precision advantage. If two cues are being combined (averaged) in a Bayes-like manner when they are both available, judgements should be more precise with both cues available than the best single cue alone. However, nearly all developmental studies so far suggest that children up to the age of 10 years do not show this effect (Adams, 2016; Burr & Gori, 2012; Dekker et al., 2015; Gori, Del Viva, Sandini, & Burr, 2008; Jovanovic & Drewing, 2014; Nardini, Bedford, & Mareschal, 2010; Nardini, Begus, & Mareschal, 2013; Nardini, Jones, Bedford, & Braddick, 2008; Petrini, Remark, Smith, & Nardini, 2014). This is despite using a broad range of tasks – uni-modal (different cues to depth but all within the visual modality) as well as multi-modal (e.g. vision and touch); two-alternative forced-choice as well as more naturalistic responses (e.g. point where it is). While this provides compelling evidence of late development, the mechanisms underlying this late development remain unclear.

Here we answer a simple but critical question about the

* Corresponding author.

E-mail address: james.negen@durham.ac.uk (J. Negen).

development of cue combination, shaping the direction of future theory. Should we think of the emergence of cue combination as a relatively unified and sudden event at about 10 years old? If so, we might favour unified explanations such as biological changes at the beginning of puberty driving its emergence. Alternatively, does cue combination emerge at meaningfully different time points for different tasks? If so, we should develop a nuanced theory that pins the emergence of different aspects of cue combination to dissociable causes contained within the earlier-emerging tasks. Clarity on this point is a key step towards a full understanding of how cue combination develops and what drives its development.

The potential applications of this knowledge range from the treatment of sensory issues in childhood over to robotics and sensory augmentation. The human brain and perceptual systems are self-organizing entities that adapt in complex ways to the needs of the environment throughout development (Stoianov & Zorzi, 2012; Van Orden, Holden, & Turvey, 2003). Understanding how the system organizes itself to implement sophisticated new algorithms – e.g. those approximating Bayesian inference (Pouget et al., 2013) – provides the necessary theoretical backdrop to understand sensory issues in atypical populations such as autism and schizophrenia, in which these computations may operate differently (Montague, Dolan, Friston, & Dayan, 2012; Wang & Krystal, 2014). Further, creating human-like adaptive capabilities in robots also requires a thorough understanding of the course of human development (Lake, Ullman, Tenenbaum, & Gershman, 2017). Finally, knowing how children learn to integrate typical cues will help us understand how to help adults integrate augmented sensory devices into their perceptual systems – a feat which adults don't perform just by exposure to a new device's use (Goeke, Planera, Finger, & König, 2016). To move towards answers that will help with these applications, we tested two hypotheses about the course of emerging cue combination skills in development, which make opposing predictions.

1.1. The Hard Limit at 10-11 hypothesis

This hypothesis states that children under 10 years old have a fundamental block, a relatively 'hard' limit, which prevents them from combining cues across all of the different tasks given. This leads to a relatively unified emergence of cue combination at 10–11 years old. This is a relatively parsimonious explanation that still fits the vast majority of the available data. To anchor this discussion, it is necessary to look in some detail at a recent study of how visual depth cues, specifically binocular disparity and relative motion, develop in middle/late childhood (Dekker et al., 2015). This study stands out as having a high number of participants ($N = 142$), sampled continuously from a relatively narrow age range (6–12 years), and with each participant completing an unusually high number of trials for a developmental sample (360). Three measures of cue combination were obtained (two behavioural and one from fMRI). All three were treated with a regression, finding a divergence from no effect after the 10th birthday but before the 11th. All three showed no significant cue combination effect in children under 10.5 years old, but a very clear and significant effect in children over 10.5 years old. This provides convergent, consistent evidence that the combination of visual depth cues is present after 11 years but not present before 10 years, at least under their task parameters.

This finding of a transition at about 10 years also converges with other findings from studies that looked at the combination of different cues. There is no finding of a bimodal precision advantage under 10 years in a wide variety of studies with different modalities and different types of judgements: self-motion and landmark cues to navigation (Nardini et al., 2008), visual and haptic cues to size and orientation (Gori et al., 2008), disparity and texture cues to slant (Nardini et al., 2010), visual and auditory cues to spatial and temporal bisection points (Burr & Gori, 2012), visual and haptic cues to object size (Jovanovic & Drewing, 2014), and finally visual and auditory cues to the number of events (Adams, 2016). Some of these studies did not test children

shortly after the age of 10 years, but those that did found a transition shortly after. For example, a partial integration model fit the majority of participants at 10–11 years (Adams, 2016) and a bimodal precision advantage was seen at 12 but not 10 years (Nardini et al., 2010).

The idea of a unified transition at about 10 years also fits with a proposed theory of why cue combination would not be seen in young children (Burr & Gori, 2012). It might actually present a subtle but important advantage. This theory suggests that failing to combine cues might make it easier to use cues for cross-calibration. In this context, cross-calibration is defined as the elimination of relative biases. A relative bias is when a given state of the world, cued in two different ways at different times, leads to different average judgements of the world state (e.g. a light and sound straight ahead are judged on average as 5 degrees left and 5 degrees right, respectively, which is a relative bias of 10 degrees). This theory also states that cross-calibration needs are most intense during the first ten years of life when the eyes, ears, and skull are developing the fastest. If these premises are true, then combining cues from an early age might bar children from effectively eliminating relative biases, leading to an extended period of disjointed multimodal perception. In that sense, it could be advantageous for children under 10 years to keep cues separate rather than combine them.

Another reason to favour a unified transition account is the possibility that the biological substrates implementing reliability-weighted averaging are not capable of accurately carrying out these computations until near the start of puberty. These mechanisms are not yet well understood, but one account proposes that they depend on the balance of excitation and inhibition over relatively large populations of neurons (Ohshiro, Angelaki, & DeAngelis, 2011). Their development is even less well understood, but it is certainly possible that it is protracted during the childhood years and into adolescence, like many aspects of cortical organisation and function (Gogtay et al., 2004).

A recent study with adults also fits with this type of explanation (Negen, Wen, Thaler, & Nardini, 2018). Adults were asked to learn new depth cues – based either on sound delay or wrist vibration – and combine these with a noisy visual cue. This means that participants had a cue-specific experience that was child-like (i.e. low), but still enjoyed the full neural maturation of adulthood. If the level of cue specific experience was more important, we would expect child-like performance from them. Instead, participants learned to combine either new cue with the noisy visual cue in just a few hours, emphasizing a strong role for general neural maturation in predicting cue combination. This is consistent with the idea that children might simply need to achieve the neural changes involved in the first decade of life before they can combine cues.

1.2. The Undiscovered task hypothesis

The alternative hypothesis is that children who are under 10 years old actually can combine cues on the right task with the right parameters, but this task has not yet been discovered yet. Under this view, children at 7–10 years old are fundamentally capable of combining cues in specific circumstances that are designed to support this. This hypothesis does not dispute previous results showing that there can be differences in perceptual strategies between adults and children under 10 years old. Instead, in short: at 7–9 years old, some tasks allow cue combination and some do not. This would mean that development is less like a hard limit at 10 years old and more like a gradient of earlier- and later-emerging cue combination performance on different tasks. This hypothesis has not yet seen any reliable support in the existing literature – by our count, only one (Nardini et al., 2013) out of 26 tests for a bimodal precision advantage below 10 years has been positive, a proportion not greater than the expected Type I error rate. However, it remains the obvious alternative to existing theory (Burr & Gori, 2012; Ernst, Rohde, & van Dam, 2016). Our central approach is to see if a new task can provide consistent evidence of cue combination in this age range.

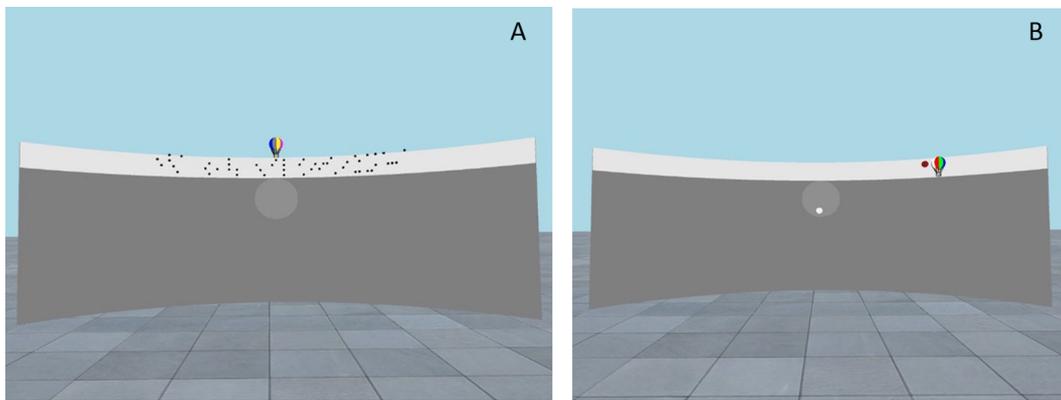


Fig. 1. Screenshots from the task. Panel A is from the Demonstration phase (described fully in *Procedure*), where participants were familiarized with the visual stimulus. Panel B shows example feedback from the Data Collection phase where the participant has placed the response (red dot) about 75% of the way to the right and the correct target ('Piccolina' in the hot air balloon) was very near, slightly further right. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

1.3. The present study

The key evidence required to favour the Undiscovered Task Hypothesis over the Hard Limit Hypothesis is a robust, repeatable finding that children under 10 years old can combine cues, standing in contrast to previous studies having shown combination only at 10 + years. To achieve this, we chose a new task with different parameters than those used in previous studies. Specifically, we chose to focus on the parameter of feedback as way of making a new task, in essence investigating how variations in feedback affect cue combination behaviour. Feedback might help participants understand the relative reliability of their perception of the two cues, which is the key to setting the correct weights (Ernst et al., 2016), and poor weight choices in this age range have been empirically demonstrated several times (Gori, Sandini, & Burr, 2012; Nardini et al., 2010). Feedback might also help reduce any relative biases. In adults, a strong relative bias prevents cue combination, since it leads to the perception of two separate causes (Shams & Beierholm, 2010). For example, it does not make sense to average the perceived positions of a light and a sound source, creating a unified estimate of their single cause, if a strong relative bias leads to the inference that the cues are from different places. The Hard Limit Hypothesis states that this should all be irrelevant and we still should not see a bimodal precision advantage in children younger than 10 years. The Undiscovered Task Hypothesis allows for the possibility that this effect will be found in this age range.

For Experiment 1a, we ran a variant of a previous spatial audio-visual cue combination task with 7–10 year old children (Gori et al., 2012). The previous study found a visual capture effect (only the visual information was used) in their version without feedback. We altered Gori et al.'s paradigm by giving feedback on single-cue trials (audio-only or visual-only), but not the audio-visual trials, forcing participants to infer how to combine the two cues only from their experience with each cue in isolation (Maloney & Mamassian, 2009). To make the feedback more meaningful, we also moved from a two-alternative forced-choice (2AFC) bisection paradigm to an absolute judgement paradigm; feedback on absolute judgements should, in principle, be more informative than feedback on 2AFC judgements. In brief, our task asked participants to judge the location of a target on a left–right horizontal axis in front of them, relying either on a noisy visual cue, a spatialized sound, or both. The target was 'hiding' behind an opaque wall and they used a computer mouse to click where they thought it was on the horizontal axis. The key analysis is whether judgements are more precise with both cues available than the same participant's best single cue. This is a result only expected under cue combination (weighted averaging).

The results of Experiment 1a were very surprising, showing a bimodal precision advantage. Given the large number of studies on this

(reviewed above), almost all of which found null results in this age range, we wanted to make sure the results were replicable (i.e. not simply a Type I error due to a large number of independent tests across many labs and studies). We therefore ran Experiment 1b, which was a direct replication of Experiment 1a, finding the same pattern of results.

For Experiment 2, we wanted to look at this phenomenon with a slightly different approach to see if we could find converging evidence. We reasoned that if children at this age are combining cues in a task like this, we should not only see a robust bimodal precision advantage, but a model comparison analysis should also generally favour a cue combination model over any single-cue models as well. Experiment 2 also had two secondary aims. The first was to see if there is an effect of feedback that cannot be explained by simple task engagement. To achieve this, we did not compare full-feedback and no-feedback conditions, as any failures to combine cues in the latter could always be because they simply find the task less engaging. Instead, we varied the type and amount of feedback across conditions in a manner that did give some feedback to all participants in order to keep motivation and engagement high. The second was to see if children can adapt to varying levels of visual reliability on a trial to trial basis. This is expected under the full remit of Bayesian Decision Theory (Maloney & Mamassian, 2009), but not under current models of multisensory learning in development (Dae, Mirian, Ahmadabadi, Brenner, & Tenenbaum, 2014; Weisswange, Rothkopf, Rodemann, & Triesch, 2011). Throughout experiments 1a, 1b, and 2, our primary motivation was to see if children at 7–10 years combine cues in our new task, which would discredit the Hard Limit Hypothesis in favour of the Undiscovered Task Hypothesis.

2. Experiments 1a and 1b

In Experiment 1a, 7- to 10-year-old children were tested on their visual and/or auditory localisation in the horizontal plane in an immersive virtual reality (VR) environment. Their task was to find a tiny virtual woman in a hot air balloon ('Piccolina') hiding behind a wall with the aid of a brief visual cue, a spatialized sound, or both (Fig. 1). To show where they perceived Piccolina to be, children used a virtual tool that they controlled using a computer mouse. This allows us to analyse the data with standard statistical tests to compare unimodal precision (audio or visual alone) and bimodal precision (both available) (Ernst et al., 2016). Based on the Hard Limit hypothesis, we would predict that their precision with both cues available would be no better than the same child's precision with their best single cue. Based on the Undiscovered Task hypothesis, we would predict that precision with both cues together would be higher on average than with the best single cue. Since the results of Experiment 1a stood out in contrast to many previous similar studies, Experiment 1b directly replicated Experiment 1a.

2.1. Method

2.1.1. Participants

77 participants were initially tested in total across Experiments 1a and 1b. 33 of these were in Experiment 1a (18 males). Four of these were excluded for failing to make estimates above chance in at least one condition (ages 7, 8, 8, and 10; 1 male), leaving 29. An ability to use each single cue (better than chance) was necessary to show that participants understood the task, and for questions about potential cue combination benefits to be meaningful. Of these 29, there were five 7-year-olds, ten 8-year-olds, ten 9-year-olds, and four 10-year-olds. This makes for a total of 25 included participants under 10 years old in Experiment 1a. The other 44 participants were in Experiment 1b (22 males). Five of these were excluded for failing to make estimates above chance in at least one condition (ages 8, 9, 9, 9, and 9; 4 males), leaving 39. Of these 39, there were four 7-year-olds, ten 8-year-olds, sixteen 9-year-olds, and nine 10-year-olds. This makes for a total of 30 included participants under 10 years old in Experiment 1b. This also means that there were a total of 55 included participants under 10 years old in both experiments together, plus an additional thirteen included 10-year-olds.

All children were enrolled in Primary Schools in County Durham, UK. All children had normal or corrected-to-normal vision and hearing. To the knowledge of the researchers, no children had been diagnosed with any perceptual or developmental disorder that might have affected task performance. This study was approved by the Psychology Department Ethics Sub-Committee at Durham University (reference: PGT2018MN1). Parents gave written informed consent.

2.1.2. Apparatus

Virtual Reality technology was used. Visual motion and positional cues in virtual space are processed similarly to how they would be in real space (Foreman, 2009), allowing for ecological validity whilst retaining a high level of experimental control. This study used WorldViz Vizard 5 software and the Oculus Rift headset. AKG K 271 MK II headphones were used with a SoundBlaster SB1240 sound card.

A simple virtual environment was created with four notable features (Fig. 1). There was a large curved wall 2 m away from where the participant sat. It had a gray fixation circle at its center. A red dot, when shown, was under the participant's control and used to indicate their estimates. The target character ('Piccolina') was a 5 cm tall woman in a 15 cm tall multi-colored hot air balloon that could either jump excitedly and clap her hands or lean over the side as if in tense anticipation of the coming events. Participants were essentially trying to estimate where Piccolina was hiding behind the top edge of the wall.

2.1.3. Stimuli

Visual stimuli consisted of an array of 64 spheres with a radius of 0.008 m, placed uniformly along the horizontal axis of the wall and randomly along the vertical axis (Fig. 1A). During 250 ms, spheres moved 1/7th of the way to the target location before disappearing. This visual cue to location was designed to be uncertain, giving a potential advantage for participants to combine the visual cue with the auditory cue when both were available. Judging the point of convergence can be described as a motion integration or coherent motion task (Burr & Thompson, 2011).

The audio stimuli consisted of white noise amplitude modulated at 10 Hz. This lasted for 300 ms. Audio stimuli were spatialized using the 'small pinnae' head-related transfer function from the CIPIC head-related transfer function (HRTF) database (Algazi, Duda, Thompson, & Avendano, 2001) for 117 different points along the wall. For these experiments, audio and visual stimuli were both accurate and aligned (i.e. no perturbations were shown). All variation in results thus reflected internal noise and biases.

Audio-spatial stimuli typically show measurable biases, even when using a speaker array and gathering data with adults (Lewald & Ehrenstein, 1998). Because HRTFs were not those of individual

participants, we might find auditory localisation to have larger biases than with a speaker array or individually-tailored HRTFs. However, because the experiment included feedback (see below), participants also had an opportunity to recalibrate for such biases. In our analysis, we checked for biases as well as precision.

2.1.4. Procedure

Children were told that they would play a virtual game of hide-and-seek. The headset and headphones were adjusted to be comfortable and ensure clear perception.

Participants were introduced to Piccolina, an animated character who would later hide from view. The researchers checked participants could see the entire virtual wall, which Piccolina would hide behind. At this point, the procedure went in three phases: Demonstration, Warmup, and Data Collection (Fig. 2).

2.1.4.1. Demonstration. A gray circle was present in the center of the screen, and a small white dot indicated the central visual point of the headset, demonstrating participants' fixation (see Fig. 1B). Participants were told they could control the white dot by moving their head, and instructed to keep it within the gray circle. If participants moved their heads during the experiment, the task would pause until the white dot was back in the gray circle. This ensured that the visual field remained the same between participants and trials.

Volume was tested. Then children were told they would see dots moving towards Piccolina, as if she held a large magnet. This was demonstrated on-screen, with visual stimuli moving from both directions towards Piccolina, centred on the white wall. The first demonstration of the visual cue was extended, allowing participants to clearly see the dots move all the way to Piccolina. Participants were then told that this was 'a little bit easy', and that they would see something a bit shorter. Children were then exposed to a second visual demonstration, showing the dots as they would appear during the task. Children were shown both demonstrations again to consolidate how the cue could be used. Piccolina then moved around the wall, moving in steps towards the far right, then far left, and back to the center-point (32 exposures). At each step, the visual stimuli were demonstrated moving towards her location. Children were instructed that this was how the dots would appear during the task, and could be used as a clue to identify her location. Piccolina then moved around the wall again, and this time children were exposed to the audio stimuli at each of the same 32 positions in the same order. Children were instructed that the sound 'following Piccolina' could also be used as a clue to identify her location.

2.1.4.2. Warmup. Piccolina then hid from view behind the top of the wall, and a single-cue trial was given (see Fig. 2-B). Children controlled a red dot using a computer mouse to report their location estimate, and a mouse-click to make their judgement. This continued for 30 trials, 15 visual-only and 15 audio-only, evenly spaced across the response area. Feedback was given on all of these trials. These data are not used in the analysis.

2.1.4.3. Data Collection. There were 70 evenly-spaced targets for this phase. Each target location was tested once with just an audio stimulus, once with just a visual stimulus, and once with both. Fig. 2-B gives the timing of each trial. The trial types were interleaved. Trial 31 was audio-only, 32 was visual-only, 33 was audio-visual, 34 was audio-only, and so on. The order of targets was randomized. Feedback was only given on trials where one cue or the other was presented (not both).

2.1.5. Constant and variable error calculations

The data were processed into constant and variable error for each participant. Cue combination is not always expected to affect constant error, but is always expected to reduce variable error (Ernst et al., 2016). To calculate these, we first need the residuals. The residuals are the response locations subtracted from the target locations (e.g. if the

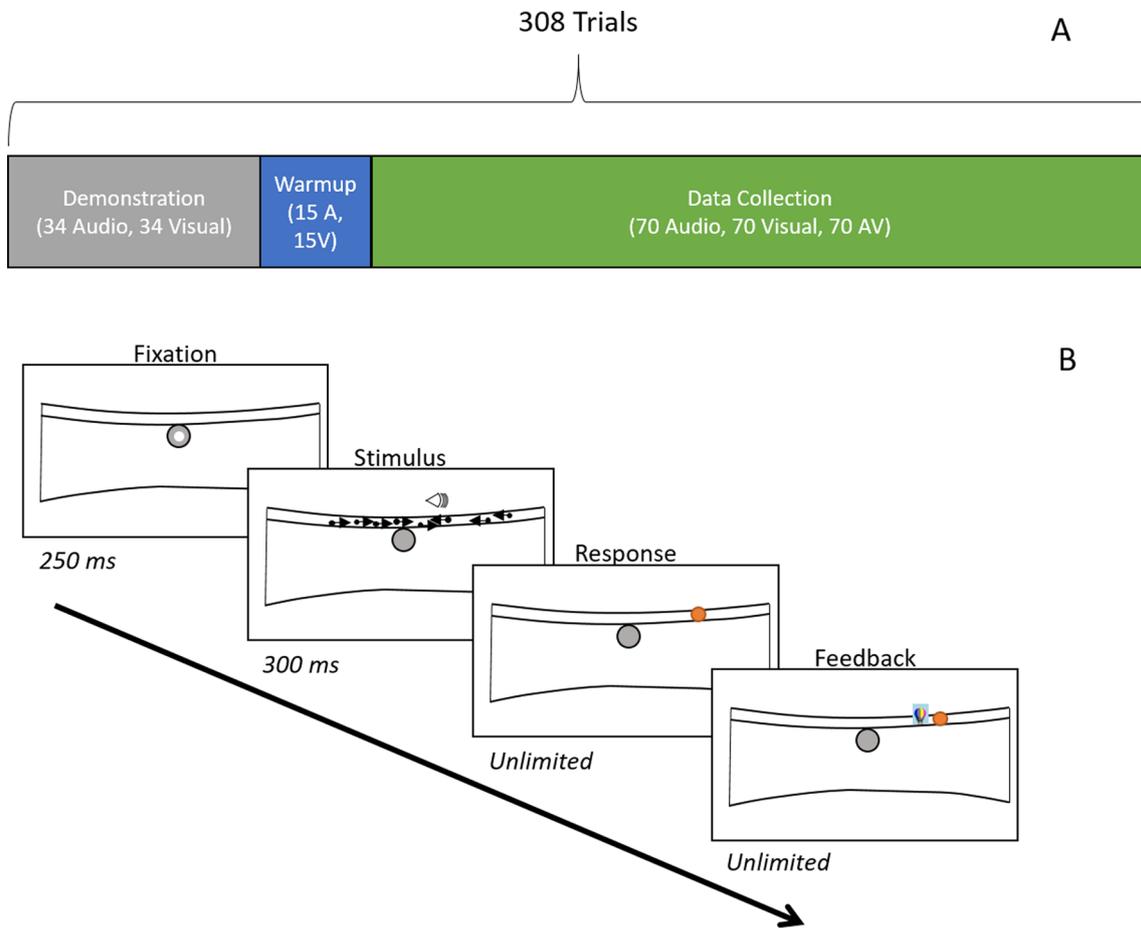


Fig. 2. Order (A) and timing (B) of the trials. For the data collection trials, after fixating for 250 ms, participants would receive a stimulus that contained audio spatial information, visual spatial information (64 dots moving briefly to a common location), or both. They would then make a response. If there was only one cue presented (audio or visual), they would then receive feedback, which remained visible until they pressed a button to begin the next trial.

target is at +25 degrees and the response at +20 degrees, then the residual is -5 degrees). The constant error is the average residual. The variable error is the variance of the residuals. This variable error represents the remaining noise after parsing away the constant error (bias), with a lower variable error corresponding to higher precision (better).

2.2. Results and discussion

Results consistently pointed towards a bimodal precision advantage and thus to cue combination. We focused first on the children under 10 years old. The main outcome measure was the variable error (VE). These VE measures were heavily skewed in many cases, such as a skewness of 2.21 for the audio-visual VE measures, so they were examined with a sign-rank analysis. (This also allows us not to be concerned with potential differences when using standard deviation, variance, or precision; they all give the same rank differences.)

The outcomes of these analyses are reported in Table 1, while the data are shown in Fig. 3. The analyses found significant within-subjects differences between VE with bimodal audio-visual cues and each participant’s best single cue in Experiment 1a (Table 1). The bimodal trials showed lower VE (average 78% of best single cue), pointing towards the presence of a bimodal precision advantage. This finding is generally considered to be the most crucial and strongest evidence of cue combination (Ernst et al., 2016) as it demonstrates that the additional (less precise) cue is being used in some way to improve the precision of estimates. Further, we did not find a difference between the audio-visual performance and the prediction of optimal performance (Table 1).

The optimal prediction is found by adding the precision from the two cues together (Ernst & Banks, 2002). This means that participants not only showed a bimodal precision effect, but that they further showed an effect that we cannot differentiate from the optimal use of the two cues together.

Experiment 1b was performed to see if these findings would replicate. They did (Table 1). There was a significantly lower VE for the bimodal trials compared to the same participant’s best single cue (on average, 73%). There was no difference between the audio-visual performance and the predicted optimal performance. This suggests that these findings are reliable and replicable.

For further analysis by age group, the data from the two experiments were pooled together for a larger sample. Each year-long age group showed a significant bimodal precision advantage in terms of VE

Table 1
Outcome measures for Experiments 1a and 1b (sign rank tests). BS = Best Single; AV = Audio-Visual; O = Optimal.

Group	Comparison	Direction	p	N	Median Diff.
1a: < 10 YO	Best Single	BS > AV	0.005	25	34.0
1a: < 10 YO	Optimal	O = AV	0.979	25	-2.90
1b: < 10 YO	Best Single	BS > AV	< 0.001	30	35.4
1b: < 10 YO	Optimal	O = AV	0.909	30	-0.01
1: 7 < Age < 8	Best Single	BS > AV	0.027	9	38.9
1: 8 < Age < 9	Best Single	BS > AV	0.006	20	42.0
1: 9 < Age < 10	Best Single	BS > AV	0.003	26	28.6
1: 10 < Age	Best Single	BS > AV	0.017	13	24.5

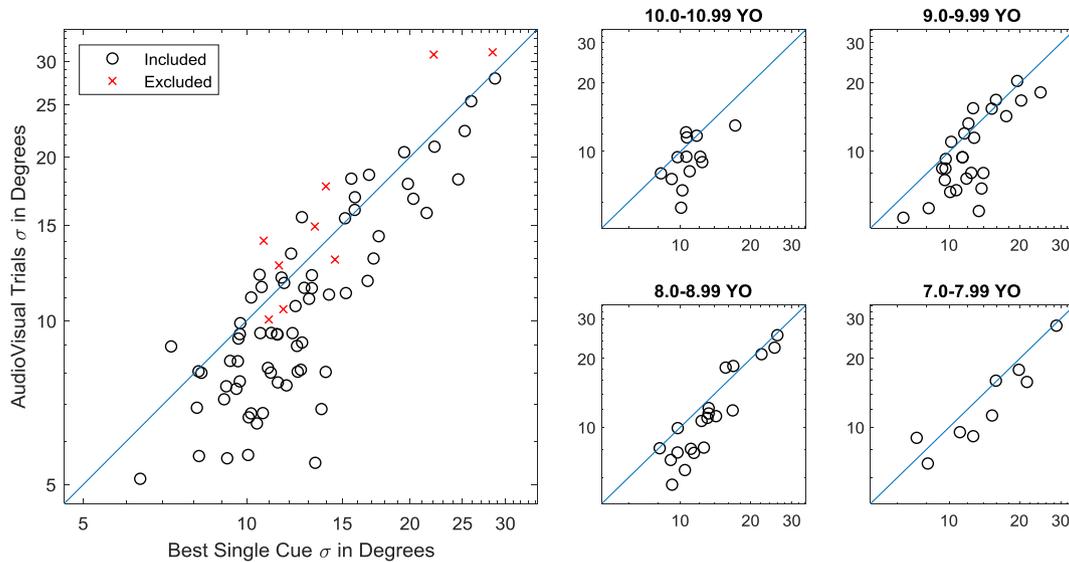


Fig. 3. Comparing the standard deviation of responses away from the targets with the best single cue (X axis) versus the two cues together (Y axis). The reference line in blue is where they are equal. Data points represent individual children. Points below the line indicate a bimodal precision advantage. Points nearer the left represent a better use of at least one cue. The left panel shows all children tested, with the right panels breaking this apart by age. Red crosses denote data from children excluded from analysis for failing to make estimates above chance in at least one condition (see Section 2.1.1. Participants). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(Table 1, Fig. 3). On average, the bimodal VE was 82%, 74%, 73% and 76% of the best single cue VE, for each age group in order. This ratio was not significantly correlated with age, $r(66) = -0.13, p = .288$.

The bimodal precision advantage was also present and significant when looking at all data collected with children under 10 years old (see Fig. 3), including the 8 children that were excluded, $N = 63, p < .001$. (One more child was also excluded, but that child was over 10 years old.) This shows that this effect does not depend on our exclusion criteria.

Biases were centered near zero (see Fig. 4) and audio-only biases were comparable in magnitude with previous reports of typical adult biases for audio-spatial stimuli (Lewald & Ehrenstein, 1998). This suggests that our use of HRTFs through headphones was not a major issue. Further, using the VEs, we can estimate the weight that would be optimal to give to each cue during audio-visual trials (weighted by precision, which is $1/VE$, as measured in single-cue trials). We can then predict, for each participant, the bias in the audio-visual trials if the two cues are being averaged with the optimal weights. The predicted and observed biases in the audio-visual trials were significantly correlated

(Fig. 4, right), $r(53) = 0.56, p < .001$. This is also consistent with cue combination.

To further deal with biases, we also looked at the mean squared error (MSE). This measure includes the bias, creating an overall measure of accuracy, which may be of more practical interest. It was calculated by squaring and then averaging the signed errors. (In contrast, the VE subtracts out the mean signed error.) This bimodal MSE was, on average, 90% and 88% of the best single cue MSE for each respective experiment. This also reached significance in Experiment 1a, $p = .019$, and Experiment 1b, $p = .006$, in a sign-rank test between the best single cue trials and the audio-visual trials. This suggests that participants not only increased precision, but also accuracy, with both cues available.

To see how this pattern of performance was reflected in the raw data, we show the full target and response distribution of a selected sample participant with lower audio-visual VE than either single cue VE in Fig. 5. Full data are also attached in Supplemental Materials for any further analysis desired by future researchers (Negen et al., 2019). Comparison with the identity line shows that bias is relatively minor (especially in the center of the range). The lower variability along the y axis in Audio-Visual than either single cue condition marks the bimodal precision advantage.

In summary, the results of both Experiment 1a and 1b suggests that children under 10 years old can combine cues in this task, perhaps even optimally. This is robust to a wide variety of ways of looking at the data as well – various outcome measures, experiments, and subgroups of the age range. This violates the predictions of the Hard Limit hypothesis and instead matches the predictions of the Undiscovered Task hypothesis.

While results from Experiments 1a and 1b already point towards favouring one of the two key hypotheses, matching the standard of evidence set by a previous project (Dekker et al., 2015) requires the effect to be shown independently a third time in a different kind of measure. We chose a model comparison measure as the strongest way of verifying this. If children under 10 years are combining cues, they should not only show a bimodal precision advantage, but a combination model of their data should be preferred over a model that posits the use of a single cue on each trial. Experiment 2 fills this role.

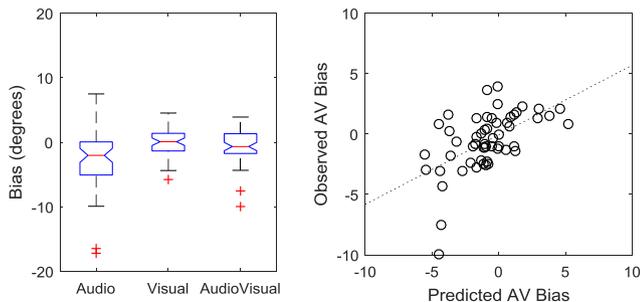


Fig. 4. Biases in the task. On the left, a box and whisker plot of observed biases in the included children under ten years of age. Notches represent a 95% confidence interval on the median. The red line is the median. The box spans from the 25th to 75th percentile. The whiskers extend two 1.5x the interquartile range or the furthest data point there. The red crosses are further data outside the whisker range. On the right, the bias in the audio-visual trials was significantly predicted by the optimal combination of the two cues. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

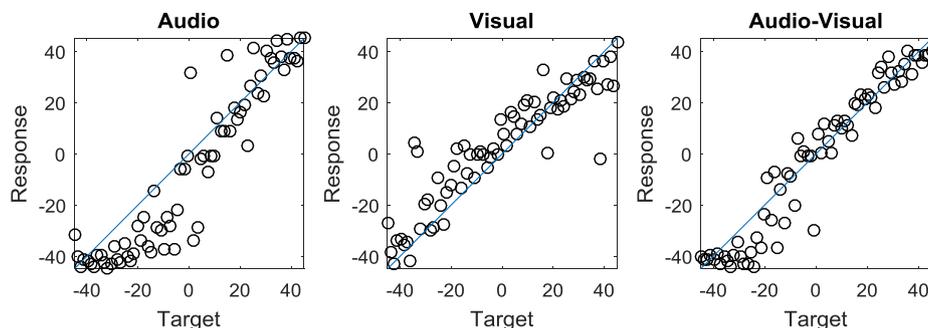


Fig. 5. Full data from a selected representative participant. The AV trials tended to be closer to the target (i.e. fall closer to a line from the bottom left to top right), whereas the response to each single cue were less precise. Points represent individual trials.

3. Experiment 2

Here, we prioritized the ability to carry out model comparison with each individual child's dataset separately. In this case, we are not primarily testing for a bimodal precision advantage with a null hypothesis of no difference and a vague alternative of some difference (though this effect was also observed). Instead, we created several generative models of the algorithm used to deal with the two sources of information, derived predictions of the bimodal trials from performance on the single-cue trials for each child and each model, and compared how well these predict the actual bimodal data for each child.

This means that we are looking at the best model for each child, not the entire group. This is a challenge with children at this age because they are unlikely to be willing and able to focus on the task for long hours or thousands of trials. Instead of raw trial numbers, we therefore used a visual stimulus with external noise. This means that the visual stimulus was extremely easy to perceive. It was clear exactly where its center was on the horizontal line. It was left visible until the participant made their response. The noise was manipulated in the way the center of the visual stimulus related to the placement of the target. Under the control of the experimenter, the target would tend to appear near the center of the visual stimulus but could vary in either direction along the horizontal line. This is external noise in the sense that the noise is applied to the stimulus by the experimenter (Macmillan & Creelman, 2004). Internal noise is when the stimulus could be used perfectly in principle but performance is limited by noise intrinsic to the participant. The use of external noise increases statistical power since we can use the known placement of the visual cue to create narrower predictions for all of the models that involve the visual cue.

We also wanted to look more at the role played by feedback. We did not want to have a condition without feedback as this may simply not be as motivating. However, we could instead manipulate if they received feedback on single-cue trials (like Experiments 1a-b), the audio-visual trials, or both. Feedback on the audio-visual trials might be more helpful than single-cue trials because it makes it the most obvious when cue combination would have led to a more accurate response, and could potentially train a cue combination strategy. On the other hand, feedback on single-cue trials was already very effective in Experiments 1a and 1b. It is also possible that learning relative to both cues at the same time is difficult for young participants. In any case, an effect where differences in feedback lead to different rates of cue combination behaviour would support the general idea that feedback is an important variable for predicting the combination of cues.

Further, we wanted to see if participants could stretch beyond the predictions shown by the experiment above. The full remit of Bayesian Decision Theory predicts not just a precision advantage for combination of cues, but also that this should persist when the relative reliability of the two cues changes – even trial to trial. To test this, we allowed the precision of the visual cue to vary randomly within a range around the running estimate of the audio cue's reliability for each participant. This

also helps ensure that the reliabilities will tend to be reasonably matched for most trials, which is also important for statistical power (Ernst et al., 2016).

3.1. Method

3.1.1. Participants

Eighty-three participants were tested, some in the lab ($N = 22$) and the rest in a local school. Of these, 16 were excluded for failing to show a significant correlation between targets and responses in the audio-only, visual-only, or audio-visual trials, leaving 67 participants. Ages ranged from 7 years and 0 months to 9 years and 11 months (17, 27, and 23 respectively in each year range). The ages were distributed evenly among the three conditions, with each having an average participant age of 8 years and 4 months. This study was also approved by the Psychology Department Ethics Sub-Committee at Durham University (reference: 16–07 Multisensory Learning). Parents gave written informed consent.

3.1.2. Apparatus and stimuli

The apparatus was the same for participants tested in the lab and in schools in terms of display equipment, headphones, and environment, but in the lab setting we gave them a virtual laser pointer as a response device instead of a mouse. In both cases, the audio stimulus was the same as Experiments 1a and 1b. The virtual environment was also the same as the previous experiments, except for the visual stimulus.

The visual stimulus was a display of a normal distribution placed over the wall (see Fig. 6). The targets were generated by actually drawing from this distribution and re-drawing if this fell off the ends of the wall. This means that the visual cue went from having internal noise to external noise for this experiment. The placement of the visual cue's center varied randomly (uniformly) across the wall. Further, crucially, the variance of the visual cue varied trial-to-trial from 75% to 125% of the running estimate of their audio variance. This was signalled by drawing the visual cue wider or narrower on each trial. In essence, this asked participants to look at the blue curve and extract both a mean and a variance. This challenged participants to not only combine the cues, but to do so with varying weights across the experimental session.

3.1.3. Procedure

Procedure for this experiment was very similar to Experiment 1a and 1b but had two notable differences. First, the feedback varied by condition between subjects: all trials (feedback on audio, visual, and audio-visual), single-cue only trials (feedback on audio, visual), or audio-visual trials. To make the feedback amount similar in the single-cue feedback condition and the audio-visual feedback condition, the participants in the audio-visual feedback condition experienced twice the number of audio-visual trials (rotating audio, audio-visual, visual, audio-visual).

Second, the demonstrations were altered. The audio demonstration

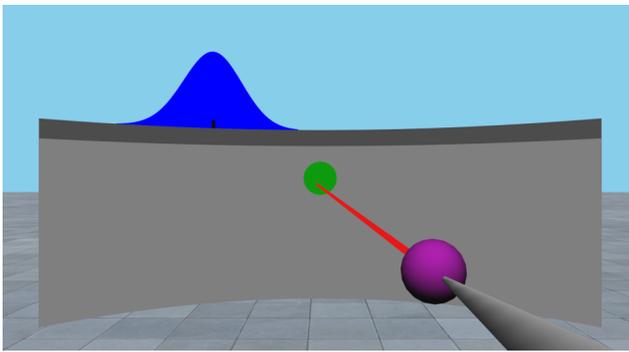


Fig. 6. Screenshot of the visual stimulus and the alternative response device used for in-lab testing. The blue curve on top of the wall is a normal distribution with a certain mean and a certain variance that changes trial to trial. The target is most likely to appear directly under its center, becoming less likely to the left and the right, exactly in proportion with the depicted height of the blue curve. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

was shortened to 16 trials. The visual demonstration was altered to be three stages of 16 draws each. The first draw was from a central, medium-variance visual cue. The second was from a leftwards large-variance visual cue. The third was from a rightwards low-variance visual cue. This was intended to introduce the idea that the reliability of the visual cue could change and that the highest point on the visual cue indicated the most likely target.

3.1.4. Models and model comparison

We considered six different models as potential candidates for explaining how each child behaved in the audio-visual trials. Each model’s mathematical implementation was derived as much as possible from the existing literature. In non-technical terms, we used each child’s data in visual-only and audio-only trials to calibrate what to expect across the different models. For example, the audio-only model treats audio-visual trials as if they were also just audio-only trials (i.e. the child just ignores the visual stimulus). To see what we expect to happen on those audio-visual trials, if that model is correct, we use the data from trials that were actually audio-only. We essentially expect those two conditions to look similar in terms of bias and noise. The other models have different strategies for the audio-visual trials, leading to different predictions. The process as a whole is designed to find the model that is best at using the single-cue trials to predict what will happen in the audio-visual trials. We describe this process in the proper technical terms in Section 3.1.4.7, but first give a description of the various models.

At a mathematical level, each model is essentially a way of taking in certain parameters and then predicting a distribution of responses on each audio-visual trial. The top of Fig. 7 gives some example parameter settings. Each panel of Fig. 7 shows what a different model would predict for the distribution of responses for an audio-visual trial. The following six subsections explain each model individually. In each subsection, the top paragraph explains the concept of the model and the bottom paragraph describes the calculations involved.

3.1.4.1. Optimal combination. In this model, the auditory and visual estimates are weighted by their relative reliability and averaged. This represents the best possible use of the two cues for maximising precision around the correct target over a large number of trials (Ernst & Banks, 2002). Note that under this model, the weights change trial-to-trial based on the varying reliability, i.e. varying external noise, of the visual cue (c.f. single weighting below).

In the example (Fig. 7), the visual cue is more reliable than the audio cue. This leads to most responses being closer to the visual cue than the audio cue. Using standard deviations of 10 and 15 degrees (for

- Visual cue
 - Centered at +5°
 - SD of 10° externally
 - No bias
- Audio cue
 - Played at +20°
 - SD of 15° in perception
 - No bias
- Other model parameters
 - Switching $C_V = 50\%$
 - Fixed Weight $W_V = 0.5$
 - Biased Weight $E = 1.5$
 - Visual-Only $F = .2$

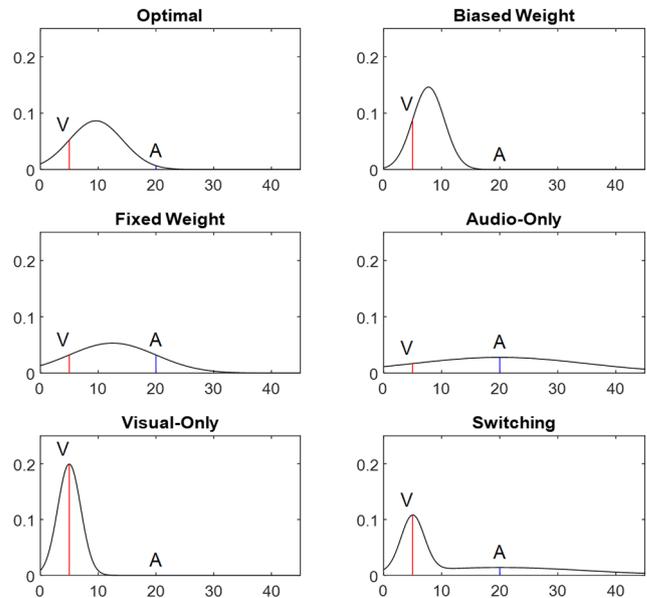


Fig. 7. An example of each model predicting audio-visual data by using the parameters at the top. The curves are probability density functions. Their height at any given point gives the probability of recording a response there, given these specific parameters. The most crucial feature is that the optimal, biased weight, and fixed weight models use both cues on every trial. This allows us to predict responses that mostly fall in between the two cues. The audio-only, visual-only, and switching models use only one cue on any given trial. This leads to distributions that are not particularly concentrated in between the two cues. The text in 3.1.4.1 through 3.1.4.6 give further details on each model. See Table 2 for the equations used to find the mean and standard deviations for the displays here.

visual and audio respectively), we calculate a visual weight of $10^{-2} / (10^{-2} + 15^{-2}) = 0.69$. We predict that the most likely response will be a weighted average of their placements (5 and 20 degrees): $5 * 0.69 + 20 * (1 - 0.69) = 9.6$ degrees. However, there is some variance left. We know exactly where the visual cue was placed, but we do not know exactly where the participant will perceive the audio cue. Therefore, the prediction for their weighted average still has a standard deviation of $(1 - 0.69) * 15 = 4.6$ degrees.

3.1.4.2. Biased weight. This is much like optimal combination, except the weights can be systematically biased. Depending on the parameter setting, it can either under-weight or over-weight the visual cue. One way to think of it is as follows: this model has a difference between the visual cue’s actual reliability and its perceived reliability. The actual reliability is multiplied by a certain number to get the perceived reliability, then the perceived reliability is used to find the weights. If that multiplicative factor is above one, both cues are still used but the visual cue is given too much weight. Just like the optimal model, the weights change trial-to-trial with the changing visual reliability (i.e. varying external noise).

In the example, the visual cue is more reliable than the audio cue and there is an additional bias towards the visual cue. Specifically, in the example, we have chosen to illustrate what happens when the perceived precision of the visual cue is $E = 1.5$ times higher than its actual precision. We do the same calculations as the optimal model, but

use a value of $10^{-2} * 1.5$ for the visual cue's precision instead of 10^{-2} . This leads to a 0.81 visual weight. The most likely response is at 7.7 degrees, which is closer to the visual cue than the optimal model. Since the audio cue is less involved, the prediction has a lower standard deviation at $15 * 0.19 = 2.7$ degrees. If we instead set E to some value below one, like $E = 0.5$, then the prediction would instead over-weight the audio cue and become wider.

3.1.4.3. Fixed weight. This is also a form of combination model, but it is unresponsive to the relative precision of the two cues. A single fixed weight for the visual cue is set between zero and one and applied for all trials.

In the example, we have chosen to illustrate what happens when $W_V = 0.5$, i.e. when the visual weight is 50%. This is lower than the optimal (69%). This means that more of the responses tend to be further from the visual cue and closer to the audio cue. The most common response is 50% of the way between the two cues at $5 * 0.5 + 20 * 0.5 = 12.25$ degrees. The audio cue is more involved than in the optimal, so the prediction has a larger standard deviation at $15 * 0.5 = 7.5$ degrees.

3.1.4.4. Audio only. The participant just uses the audio cue. This is essentially just modelling the audio-visual trials as if they were additional audio-only trials.

In the example, the audio cue is placed at 20 degrees. Since there is no bias, we expect the most likely response to be at 20 degrees. This happens to be far from the visual cue. However, there is also a 15 degree standard deviation in audio perception. The prediction distribution therefore also has a standard deviation of 15 degrees, much higher than the optimal (4.6 degrees).

3.1.4.5. Visual only. The participant just uses the visual cue. This is essentially just modelling the audio-visual trials as if they were additional visual-only trials.

In the example, the visual cue is placed at 5 degrees. There is no bias, so we expect the most likely response to be at 5 degrees. This happens to be far from the audio cue. The standard deviation of responses is the standard deviation of the stimulus itself times a factor F . In our notation, $SD_{VResp} = SD_{VStim} * F$. In the example, we have chosen to illustrate what happens when $F = 0.2$. We expect the standard deviation of responses to be $10 * 0.2 = 2$ degrees since the visual cue has a standard deviation of 10. For our purposes here, it does not matter exactly why this noise exists (i.e. why they did not point exactly to the center of the visual cue). It could be some combination of motor error, a sampling or pseudo-sampling strategy, an effect of where the last visual-only target appeared, and so on. The important thing is that we model the audio-visual trials here as if they were visual-only trials, including the unclassified noise that was present in the actual visual-only trials.

3.1.4.6. Switching. The participant has a probability of choosing either the visual cue or the audio cue and just using it alone on each trial. They never use both cues on any single trial, but they do switch between the two single cues on separate trials over the course of the experiment.

This is just a mixture of the audio-only and the visual-only models. In the example, we have chosen to illustrate what happens when $C_V = 50\%$, i.e. the prediction is found by taking half of the height of the audio-only prediction and adding it to half of the height of the visual-only prediction. Given the parameters, the most likely response is at 5 degrees, centered over the visual cue. However, there is a second local mode over the audio cue. If we instead set C_V to 75%, we would take 75% of the height of the visual-only prediction and add it to 25% of the height of the audio-only prediction.

3.1.4.7. Overview of the model comparison algorithm. The model comparison procedure went through a series of steps for each

individual participant. The end result was a single selected model that performs best in terms of predicting each participant's audio-visual data from their audio-only data and their visual-only data. This was approached by Bayesian Markov Chain Monte Carlo. For an explanation of the theory behind this, see (Kass & Raftery, 1995), especially Section 4.2 ("Simple Monte Carlo", the exact method used here). In their notation, we need to integrate $pr(\mathbf{D}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})d\boldsymbol{\theta}$ for each model. \mathbf{D} is the audio-visual data, $\boldsymbol{\theta}$ are the parameters that the models use, and π is either the distribution derived from the single-cue data or a prior distribution. We do this by sampling $\boldsymbol{\theta}^{(i)}$ from $\pi(\boldsymbol{\theta})$ and averaging $pr(\mathbf{D}|\boldsymbol{\theta}^{(i)})$. This happens in four steps.

First, audio-only parameters were sampled using the audio-only data. This was done in the style of a linear regression. Each response was modelled as a draw from a truncated normal. Truncations were placed at $+45$ and -45 degrees since the participant was not allowed to respond outside this range. There was a slope parameter S_A , a bias parameter B_A , and a standard deviation SD_A . The mean was the audio stimulus's position times the slope, plus the bias. Samples were taken from the posterior distribution via slice sampling with flat priors. The samples tended to be near the best fit, but also varied somewhat around the best fit, reflecting the uncertainty in their exact correct values. For a tutorial that explains posterior distributions and this type of sampling method, please see (Lee & Wagenmakers, 2013).

Second, the visual-only parameters were sampled using the visual-only data. The approach was the same as for the audio-only parameters in terms of slope and bias. The standard deviation of responses, SD_{VResp} , was modelled as proportional to the standard deviation of the visual stimulus, SD_{VStim} : $SD_{VResp} = F * SD_{VStim}$. This was done because responses varied further from the center of the visual cue when it was wider.

At this point, all fitting of the data has been concluded and was based solely on audio-only and visual-only data. The audio-visual data have not yet been used.

Third, any additional parameters were drawn from their prior distributions. This specific step is completely independent from all of the data. The parameters were a bias in the weights for the biased weight model (E), the weight for the single-weight model (W_V), and the rate of choosing the visual cue for the switching model (C_V). Each of these is a single parameter for all of the audio-visual data for a single participant (not a copy for each trial). In the notation of Kass and Raftery (1995), all nine parameters sampled so far are a single vector $\boldsymbol{\theta}^{(i)}$ consisting of S_A , S_V , B_A , B_V , SD_A , F , E , W_V , and C_V . We next calculate $pr(\mathbf{D}|\boldsymbol{\theta}^{(i)})$ for each model.

Fourth, all sampled parameters are fed into each model so that we can see how well they predict the audio-visual data. Specifically, we are looking for the highest joint probability of the actual responses in audio-visual trials. In all cases except switching, the audio-visual data are modelled as drawn from a truncated normal. Table 2 shows how the different models use the different parts of $\boldsymbol{\theta}^{(i)}$ to predict the mean (M_{AV}) and standard deviation (SD_{AV}) of the responses. The joint probability is found by multiplying all the probabilities of the individual responses. Each individual probability is the density of the truncated normal pdf evaluated at the response for a given trial. In Fig. 7, this means finding the response on the x-axis and then finding the height of the curve at that location. For the switching model, we instead average together the individual probabilities from the audio-only and visual-only models. Specifically, $C_V \sim \text{Uniform}(0,1)$, $C_A = 1 - C_V$, and $Pr_{AV} = Pr_V * C_V + Pr_A * C_A$. Here, Pr_V is the probability assigned to each response by the visual-only model and Pr_A is for audio.

These first four steps were repeated 30,000 times and the first 5000 were discarded as burn-in. That was repeated independently in four chains, resulting in 600,000 joint probabilities (100,000 per model). These joint probabilities were averaged so that each model had one grand probability of the audio-visual data. The model that assigned the highest grand probability to the audio-visual data was selected. This was the preferred model in the Bayesian sense: integrating over its

Table 2
Reference for model parameters.

Model	Mean	Standard deviation
Optimal	$W_V = P_{Vstim}/(P_A + P_{Vstim})$ $W_A = 1 - W_V$ $M_A = A * S_A + B_A$ $M_V = V * S_V + B_V$ $M_{AV} = W_A * M_A + W_V * M_V$	$SD_{AV} = W_A * SD_A$
Biased	$E \sim \text{Gamma}(2,1)$ $W_V = P_{Vstim} * E / (P_A + P_{Vstim} * E)$ W_A, M_A, M_V, M_{AV} same as optimal	$SD_{AV} = W_A * SD_A$
Single	$W_V \sim \text{Beta}(2,2)$ W_A, M_A, M_V, M_{AV} same as optimal	$SD_{AV} = W_A * SD_A$
Visual	$M_{AV} = V * S_V + B_V$	$SD_{AV} = SD_{VResp} = F * SD_{Vstim}$
Audio	$M_{AV} = A * S_A + B_A$	$SD_{AV} = SD_A$

Note. P_A refers to the precision of responses with the audio-only trials. Precision is $1/\text{variance}$. P_{Vstim} is the precision of the visual stimulus. A is the placement of the audio cue. V is the visual cue's center. S_A is the slope during audio-only trials. B_A is the bias during audio-only trials. S_V and B_V are the same but for visual-only trials. SD_{VResp} is the standard deviation of responses around the center of the visual stimuli on visual-only trials. In the end, each response is assigned a probability based on the truncated normal distribution with a mean of M_{AV} , a standard deviation of SD_{AV} , and truncations at -45 and $+45$ degrees. The joint probability is the product over all audio-visual trials completed by the participant.

uncertainty, informed by the audio-only and visual-only data, it has predicted the audio-visual data better than the other models.

3.2. Results and discussion

For 34 participants (out of 67), the best fitting model had a pairwise Bayes factor of at least 10 times above the second-best model. Fig. 8 gives the counts of the best fitting models by feedback condition. Overall, results suggest that the majority of participants were combining cues (in total 65% of children combined) either optimally (33%) or with biased weights (30%), and with one child best fitted by the fixed weight model. Strategies relying on one cue on each trial (i.e. no combination) were rarer, with 19% switching between trials and 16% just using the visual cue. While this highlights heterogeneity in this age range, it still speaks strongly to the idea that cue combination in this task can be done by children under 10 years.

To test the effect of conditions with a chi-square, the counts of the three combination models (optimal, biased, fixed) were collapsed, as were the counts of the single-cue use models (switching, visual-only,

audio-only), because many individual cells had zero data points within them. A significant association between feedback condition and model category was found, $N = 67, \chi^2(2) = 10.38, p = 0.006$, suggesting that the different levels of feedback led to different rates of cue combination. The condition with the most models in the combination category was the single-cue feedback condition (88% combining optimally or sub-optimally). This was lower in the conditions with feedback on all trials (64%), followed by feedback only on audio-visual trials (41%). This is still consistent with the hypothesis that this age range is capable of combining cues in this variation of the task. However, it does suggest that feedback on single-cue trials is generally more helpful in aiding cue combination than feedback on dual-cue trials. This difference emphasizes a role for the feedback in predicting cue combination.

Another notable finding is that we found only one child best fit by the fixed weight model and none best fit by the audio-only model. This might indicate that for children at this age, the major issue is not (a) being incapable of having their responses change based on changing relative reliability or (b) allowing audio capture for a spatial judgement. Instead, the major issues seem to be (1) learning the correct weights for the two cues, (2) deciding to integrate them both rather than switch back and forth between, and (3) using an audio cue at all for a spatial judgement – in a sense, the most extreme example of incorrect weights. In addition, the preference for the visual-only model over the audio-only model converges with a previous project (Gori et al., 2012), so we now have consistent evidence that visual capture is a major issue for spatial judgements with audio-visual cues in development.

We also did two things to make sure that the models were working as intended. First, we generated 100 datasets of 210 trials from each model and ran them through the comparison. These simulations used a gamma distribution of noise levels in each cue that was similar to the actual child data. We found that the method recovered the generating model on 96.8% of datasets. This confirms that the models are differentiable and that the analysis method is a reliable way to identify the correct model among these. Second, we reasoned that there should be a bimodal precision advantage as seen in the previous experiments 1a and 1b if the model fits are correct. We found indeed that there was, with the bimodal trials showing lower variable error than the best single cue trials in a sign-rank test, $N = 67, p < .001$.

4. General discussion

All three experiments provided results converging on cue combination in this task in children over 7 years and under 10 years old. This can be interpreted alongside another report about the combination of motion and disparity cues to depth (Dekker et al., 2015). That study provided three independent analyses that converged on children in this same age range failing to combine cues, but instead doing so over the age of 10.5 years. This further sits within a larger literature where there have been 25 separate failures to find a bimodal precision advantage under the age of 10 years old. Taken together, our current data and previous results provide convergent evidence that cue combination emerges at different time points for different tasks. As a whole, this is compelling evidence against the idea that cue combination emerges in a unified fashion in development, instead strongly suggesting that children under 10 years old can combine cues – they just need the right task.

The model comparison results in Experiment 2 also clarify which of the proposed alternative strategies are most likely to fit performance when it is not best fit by optimal cue combination. The most common alternative is combination with the incorrect weights. We also saw many participants discarding the audio cue entirely, converging with a previous report where children in the same age range showed a similar effect (Gori et al., 2012). This suggests that it remains tempting through middle childhood to over-rely on vision for spatial judgements, sometimes even to an extreme where audition has no effect at all.

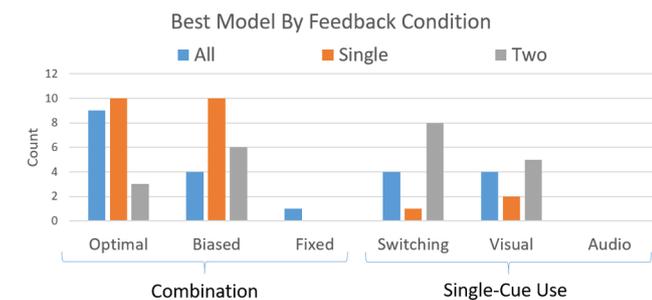


Fig. 8. The count of participants best fit by each model in each condition. Participants in the All (blue) condition received feedback on all trials. Participants in Single (orange) received feedback only on single-cue trials. Participants in Two (grey) received feedback only on trials with both cues. The majority of children were best fit by a combination model (65%), especially in the single-cue feedback condition (88%). Biased weighting of the two cues was a very common non-optimal strategy, as were switching and relying solely on the visual cue. However, few children failed to respond trial-to-trial to the varying cue weights (fixed weight) or used the audio cue exclusively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

We also found further evidence for an effect of feedback on cue combination behaviour in Experiment 2, but not the simple one that might be expected. The simplest story to tell is that children between 7 and 10 years combine cues with feedback, like our Experiments 1a-b, but do not combine cues without feedback, like a previous audio-visual study in the same age range (Gori et al., 2012) which found strong visual capture. But as our Experiment 2 shows, the full results are more nuanced: some appreciable level of cue combination behaviour was seen in all three feedback conditions, but the condition with single-cue only feedback showed the highest rate (88%) and the condition with dual-cue only feedback showed the lowest (41%). This suggests instead that single-cue feedback effectively promotes cue combination behaviour, changing the rate of cue combination observed, but not acting strictly as a binary switch.

Finally, only one child was best fit by a model with fixed weights in Experiment 2. Under this model, children do not adapt on a trial-by-trial basis to the varying reliability of the visual stimulus. They instead set one weight for the visual information and apply it regardless of the varying reliability. The result that this model was rarely selected indicates that children, once they are no longer selecting cues, quickly come to understand that the weights given to each modality must vary based on the nature of the stimulus presented (Maloney & Mamassian, 2009).

The primary finding is a key guide to the development of future theory. At the moment, papers that model the acquisition of cue combination tend to discuss emergence as a unified whole (Dae et al., 2014; Weisswange et al., 2011). Current models also do not (at least explicitly) differentiate various tasks in ways that predict their different courses of emergence. Future models should take on this challenge and attempt to understand why the same computations might emerge at different developmental points in different tasks. Further understanding of the order of emergence will let us see the crucial contours of what makes an ‘easy’ or ‘hard’ learning problem, thus gaining insight into the learning process.

4.1. Favoured explanations

This finding opens up a new theoretical depth in our understanding. With this result in hand, it is now sensible to ask what exactly makes this task different from the large body of attempts to find a similar bimodal precision advantage in this age range. In this section, we present and critically evaluate the two explanations that seem most likely to us. In the section after, we argue why some competing explanations are less compelling.

4.1.1. Feedback adjusting perceived relative reliability

Under this hypothesis, participants come into any given cue combination task in the 7–10 year old range equipped with the (optimal) reliability-weighted-averaging algorithm. However, they lack correct estimates of the relative reliability of the two cues. Without this, they default to a switching strategy, default to a cue selection strategy, or combine the cues with incorrect weights. No bimodal precision advantage obtains. In contrast, in our experiments, the feedback allows them to estimate the correct weights with reasonable accuracy and then create a bimodal precision advantage through cue combination.

The question then is why natural experience would not give children under 10 years good estimates of relative reliability. Here we think there is at least one plausible answer. The problem of estimating the reliability of a wide variety of audio and visual stimuli, in terms of their localization, may be too overwhelming for various reasons. It might place memory demands that are too high or require them to develop a fuller internal model of their own perception than they currently have. The present task, in contrast, requires them only to derive an estimate of the reliability of the subset of audio and visual cues that we presented. It seems plausible that this smaller problem is tractable but the larger issue of a good general model of own uncertainty is not.

This explanation fits well with previous research and Experiment 2. A previous study which directly assessed weights in an audio-visual task found that they are frequently far from the optimal weights in this age range (Gori et al., 2012), lending plausibility to the explanation above. Given the results by condition in Experiment 2, we also expect that feedback must play some sort of role in the correct explanation.

4.1.2. Feedback reducing bias

Under this hypothesis, biases in perception prevent cue combination and the feedback acts against that effect. Suppose we present two cues that a target is at some specific point. The participant has a particular set of relative biases. When both Cue A and Cue B signal the same point, the participant tends to perceive Cue B as signalling something +10 higher than Cue A. If the standard deviation of perceptions is well less than 10 for both cues, the participant will typically perceive the two cues to be signalling irreconcilable states of the world. Under these conditions, an optimal computation rejects the idea that the two cues have the same cause and simply uses the more reliable of the two cues in isolation (Knill, 2007; Shams & Beierholm, 2010). This kind of filtering makes perception more robust to unexpected issues, but, in this hypothetical, does so at the cost of a bimodal precision advantage.

The feedback would most likely help correct such an issue. It would provide an objective way to see that perceptions from one cue or the other (or both) are not just noisy, but on average biased in one direction or the other and thus should be corrected. This also fits well with theory regarding cross-calibration (Burr & Gori, 2012), as the single-cue feedback can serve for this calibration. This also generally fits with the results of Experiment 2, which suggests some role for feedback in the current findings. However, we are not aware of any documentation that children in this age range actually have such severe biases.

As above, it seems plausible that natural experience would not be sufficient to help children find and correct relative biases for a wide range of possible stimuli in the audio and visual domain, but that the focused task of correcting biases on these specific stimuli might be within reach.

Of course, this explanation is not exclusive with the one above – children at 7–10 may both have biases and incorrect weights that need to be adjusted for a bimodal precision advantage. The feedback here could fill both roles.

If we accept these explanations, it shifts some of the texture of what cue combination learning models need to explain. At the moment, models take on the general idea of audio-visual cue combination as essentially one task where an audio stimulus in horizontal location has the same reliability as all further possible audio stimuli in the same place (Dae et al., 2014; Weisswange et al., 2011). The estimated reliability parameter for each modality can adapt over long times (thousands of trials), but there are not, for example, separate reliability estimates for a dark room versus a brightly lit room. The way that we are discussing it here, learning the relative reliability of these specific audio-visual stimuli is not the same as developing an overall model of how reliable one’s own auditory and visual system are under different circumstances. Current models do not make it obvious how that kind of segmentation would occur in cognition.

4.2. Unlikely explanations

In this section, we argue why several additional tempting options for explaining the current results are unlikely.

4.2.1. Exposure frequency

At the moment, current models do have one implicit aspect that could differentiate some pairs of cues (Dae et al., 2014; Weisswange et al., 2011). Specifically, they assume that learning is mainly a function of the number of joint exposures to the cues, plus some noise. Therefore, one tempting possibility is that different cue pairs occur with different frequencies in naturalistic settings and that children learn to

combine the most common ones first. In a broad sense, we do know from deprived rearing tasks that this is a relevant variable – for example, cats raised in deprivation of co-varying spatial sounds and lights will fail to show audio-visual localization effects that are consistent with cue combination (Xu, Yu, Rowland, Stanford, & Stein, 2014).

However, this explanation faces some difficulties. First, it is not clear that audio-visual localization is something that children can practice more frequently than visual depth (Dekker et al., 2015), and it fails to explain why the other audio-visual study (Gori et al., 2012) did not find a bimodal precision advantage. Second, modelling the acquisition of cue combination as purely a function of cue exposure runs aground of a recent finding in adults (Negen, Wen, Thaler, & Nardini, 2017). In a 5-h task, adults were able to combine a new audio cue to depth with a familiar but noisy visual cue to depth. Therefore, something besides experience is also a strong predictor of learning to combine cues; otherwise, children would combine things like visual depth cues well before 10 years of age. Perhaps then, it is best to think of a certain level of exposure frequency as being necessary but not sufficient to induce cue combination in a specific task.

4.2.2. Better task engagement in our task than previous tasks

Experiment 2's results suggest that this is, at least, not the entire explanation for results here. Participants still varied in how likely they were to combine cues across conditions that all had feedback, all had a social entity, and all took place in virtual reality. Large differences in the level of engagement seem unlikely. It also seems unlikely that all of the previous projects failed to reach a high level of task engagement. Like exposure frequency, we interpret this as being necessary but not sufficient.

4.2.3. Feedback providing participants with a Look-Up Table

This possibility has been discussed by previous theory at length (Maloney & Mamassian, 2009). Under this hypothesis, training allows participants to associate each specific pairing of single-cue perceptions with a specific response. In theory, the observer can then make accurate responses by recalling the best response to each stimulus pairing from past experience (i.e. using a “look-up table”), and could even show a bimodal precision advantage with this algorithm. However, this is not a possibility for the specific design here: in Experiments 1a, 1b, and the “Single” condition of Experiment 2, we gave no feedback on trials where they had both cues available, yet participants still showed evidence of cue combination. Even beyond that, participants who were only given feedback on trials with both cues actually became less likely to combine them.

4.2.4. Feedback teaching participants the correct algorithm

Along similar lines, a model learner could keep track of the outcomes of their perceptual decisions and see which of several algorithms leads to the best performance. In other words, if you look over the models we compared in Experiment 2, one might imagine a child doing the same kind of thing in their own mind – but instead of an external experimenter trying to decipher their preferred strategy, the participant is trying to decipher the strategy that will lead to the best performance. This is again a poor fit for the current results for the same reason as above, the lack of feedback with both cues available. Most participants would not have anything to fit this against, and the ones who did were less likely to combine cues.

4.3. Related issues

Given recent findings in adults, we should also point out a related quandary about development that is also very open. While this study is the first to show cue combination in children under 10 years, it is perhaps not actually unique in showing that the combination of some cues follows a radically different developmental path from visual depth cues. Specifically, even adults don't combine certain cues to motion

(e.g., Soyka, de Winkel, Barnett-Cowan, Groen, & Bühlhoff, 2011), or at least vary in whether or not they combine them (e.g., de Winkel et al., 2013). This is somewhat difficult to explain under current models of multisensory learning (Daee et al., 2014; Weisswange et al., 2011), which should always converge on cue combination for any two common cues to a common judgement. Thus, a truly accurate model of how cue combination is acquired must explain why this task is learned earlier, unreinforced visual depth is learned later, and things like angular self-motion might not necessarily be learned at all.

The study of cue combination is a part of the larger realm of Bayesian Decision Theory. This theory, which suggests that perception is fundamentally well-represented by a model of internal Bayesian statistics, creates a wide range of predictions (though c.f. Rahnev & Denison, 2017). We have taken a step in this direction by not only showing a bimodal precision advantage, but showing that a model where children re-weight cues trial-to-trial fits better more frequently than one with a single fixed weight. This shows some of the re-weighting predictions to be sensible in this age range, at least for the right task. A full examination of the broader theory would require some additional work: when and how children learn to learn prior distributions (Körding & Wolpert, 2004), learn to maximize gain functions rationally (Körding & Wolpert, 2006), learn to calibrate confidence (Sporer, Penrod, Read, & Cutler, 1995), and acquire a causal inference filter (Shams & Beierholm, 2010). Compared to cue combination, relatively little is known about how each of these develop. Understanding that full breadth is important for a complete theory of multisensory perceptual development.

4.4. Towards applications in autism, robotics, and sensory augmentation

This task and finding could be used in the future as a comparison point to work towards a deeper understanding of sensory atypicalities such as those associated with autism or schizophrenia. Various theories for various conditions have posited specific differences involving the integration of information from different modalities (e.g. Baum, Stevenson, & Wallace, 2015), while others have focused on alternatives like the learning of prior information (Lawson, Mathys, & Rees, 2017). The present study has been able to take a closer look at typical development and discover a multisensory integration capacity that was not known before. It could be informative to explore if the bimodal precision advantage measured here is associated with neurodiversity in any systematic way. For example, the feedback might be used in different ways by different populations.

In terms of robotics, our results point towards an interesting suggestion for self-organization. We posit that the children in our task were not generally capable of solving any arbitrary audio-visual cue combination problem (Gori et al., 2012), but that the feedback given here allowed them to create a temporary working solution for a narrow range of audio-visual stimuli (specifically those used in the task). This might be part of the path forward for a robot attempting to adapt to different environments as well (Lake et al., 2017). It might be most effective, or at least most human-like, if a robot were to develop smaller solutions to more restricted and common cue combination problems and then to generalize much later from that set of smaller solutions.

In terms of sensory augmentation, two studies have been done that stand in stark contrast. One found that adults can integrate augmented sensory skills with their typical repertoire (Negen, Wen, Thaler, & Nardini, 2017), but the other found that they did not (Goeke et al., 2016). The study with feedback showed integration, but not the study without feedback. The present study shows that in young participants, feedback can be an important variable. This suggests an overall pattern that remains important into adulthood, with a strong emphasis on frequent feedback in order to help participants learn how to integrate a sensory skill. It may be that single-cue feedback is particularly effective for adults learning to integrate a new sensory skill as well.

4.5. Conclusion

Across three independent experiments, we found evidence that children between 7 and 10 years old can combine audio-visual cues to location in a way that is more precise than either single cue alone. This establishes that cue combination is possible before 10 years of age in some tasks. Comparing with previous research, this further establishes that cue combination emerges at different points in development for different tasks. We propose that this is most likely to be due to the feedback given during the task, rather than the other details of the task chosen. This interpretation suggests specifically that children between 7 and 10 years are able to combine cues with some intensive experience to calibrate the two cues (correcting biases and/or assessing relative reliability), but will not do the same if forced to rely on their natural experience with them. Current models of the developmental process do not account for this and should be extended to create a better theory of how cue combination develops.

Declaration of Competing Interest

None.

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Appendix A. Supplementary material

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.cognition.2019.104014>.

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